

Contents

Why Equations? / viii

- 1 The squaw on the hippopotamus / 1**
Pythagoras's Theorem
- 2 Shortening the proceedings / 21**
Logarithms
- 3 Ghosts of departed quantities / 35**
Calculus
- 4 The system of the world / 53**
Newton's Law of Gravity
- 5 Portent of the ideal world / 73**
The Square Root of Minus One
- 6 Much ado about knotting / 89**
Euler's Formula for Polyhedra
- 7 Patterns of chance / 107**
Normal Distribution
- 8 Good vibrations / 131**
Wave Equation
- 9 Ripples and blips / 149**
Fourier Transform
- 10 The ascent of humanity / 165**
Navier–Stokes Equation
- 11 Waves in the ether / 179**
Maxwell's Equations
- 12 Law and disorder / 195**
Second Law of Thermodynamics

13 One thing is absolute / 217

Relativity

14 Quantum weirdness / 245

Schrödinger's Equation

15 Codes, communications, and computers / 265

Information Theory

16 The imbalance of nature / 283

Chaos Theory

17 The Midas formula / 295

Black–Scholes Equation

Where Next? / 317

Notes / 321

Illustration Credits / 330

Index / 331

To avoide the tedious repetition of these wordes: is
equalle to: I will sette as I doe often in woorke use, a paire of
paralleles, or gemowe lines of one lengthe: =====, bicause
noe .2. thynges, can be moare equalle.

Robert Recorde, *The Whetstone of Witte*, 1557

Why Equations?

Equations are the lifeblood of mathematics, science, and technology. Without them, our world would not exist in its present form. However, equations have a reputation for being scary: Stephen Hawking's publishers told him that every equation would halve the sales of *A Brief History of Time*, but then they ignored their own advice and allowed him to include $E = mc^2$ when cutting it out would allegedly have sold another 10 million copies. I'm on Hawking's side. Equations are too important to be hidden away. But his publishers had a point too: equations are formal and austere, they look complicated, and even those of us who love equations can be put off if we are bombarded with them.

In this book, I have an excuse. Since it's *about* equations, I can no more avoid including them than I could write a book about mountaineering without using the word 'mountain'. I want to convince you that equations have played a vital part in creating today's world, from mapmaking to satnav, from music to television, from discovering America to exploring the moons of Jupiter. Fortunately, you don't need to be a rocket scientist to appreciate the poetry and beauty of a good, significant equation.

There are two kinds of equations in mathematics, which on the surface look very similar. One kind presents relations between various mathematical quantities: the task is to prove the equation is true. The other kind provides information about an unknown quantity, and the mathematician's task is to *solve* it – to make the unknown known. The distinction is not clear-cut, because sometimes the same equation can be used in both ways, but it's a useful guideline. You will find both kinds here.

Equations in pure mathematics are generally of the first kind: they reveal deep and beautiful patterns and regularities. They are valid because, given our basic assumptions about the logical structure of mathematics, there is no alternative. Pythagoras's theorem, which is an equation expressed in the language of geometry, is an example. If you accept Euclid's basic assumptions about geometry, then Pythagoras's theorem is *true*.

Equations in applied mathematics and mathematical physics are usually of the second kind. They encode information about the real

world; they express properties of the universe that could in principle have been very different. Newton's law of gravity is a good example. It tells us how the attractive force between two bodies depends on their masses, and how far apart they are. Solving the resulting equations tells us how the planets orbit the Sun, or how to design a trajectory for a space probe. But Newton's law isn't a mathematical theorem; it's true for physical reasons, it fits observations. The law of gravity might have been different. Indeed, it *is* different: Einstein's general theory of relativity improves on Newton by fitting some observations better, while not messing up those where we already know Newton's law does a good job.

The course of human history has been redirected, time and time again, by an equation. Equations have hidden powers. They reveal the innermost secrets of nature. This is not the traditional way for historians to organise the rise and fall of civilisations. Kings and queens and wars and natural disasters abound in the history books, but equations are thin on the ground. This is unfair. In Victorian times, Michael Faraday was demonstrating connections between magnetism and electricity to audiences at the Royal Institution in London. Allegedly, Prime Minister William Gladstone asked whether anything of practical consequence would come from it. It is said (on the basis of very little actual evidence, but why ruin a nice story?) that Faraday replied: 'Yes, sir. One day you will tax it.' If he did say that, he was right. James Clerk Maxwell transformed early experimental observations and empirical laws about magnetism and electricity into a system of equations for electromagnetism. Among the many consequences were radio, radar, and television.

An equation derives its power from a simple source. It tells us that two calculations, which appear different, have the same answer. The key symbol is the equals sign, $=$. The origins of most mathematical symbols are either lost in the mists of antiquity, or are so recent that there is no doubt where they came from. The equals sign is unusual because it dates back more than 450 years, yet we not only know who invented it, we even know *why*. The inventor was Robert Recorde, in 1557, in *The Whetstone of Witte*. He used two parallel lines (he used an obsolete word *gemowe*, meaning 'twin') to avoid tedious repetition of the words 'is equal to'. He chose that symbol because 'no two things can be more equal'. Recorde chose well. His symbol has remained in use for 450 years.

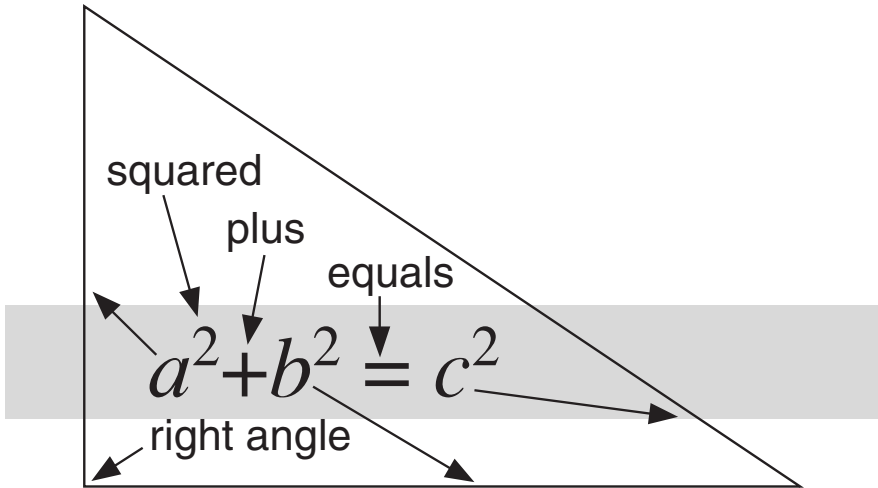
The power of equations lies in the philosophically difficult correspondence between mathematics, a collective creation of human minds, and an external physical reality. Equations model deep patterns in the outside world. By learning to value equations, and to read the stories

they tell, we can uncover vital features of the world around us. In principle, there might be other ways to achieve the same result. Many people prefer words to symbols; language, too, gives us power over our surroundings. But the verdict of science and technology is that words are too imprecise, and too limited, to provide an effective route to the deeper aspects of reality. They are too coloured by human-level assumptions. Words alone can't provide the essential insights.

Equations can. They have been a prime mover in human civilisation for thousands of years. Throughout history, equations have been pulling the strings of society. Tucked away behind the scenes, to be sure – but the influence was there, whether it was noticed or not. This is the story of the ascent of humanity, told through 17 equations.

1

The squaw on the hippopotamus Pythagoras's Theorem



What does it tell us?

How the three sides of a right-angled triangle are related.

Why is that important?

It provides a vital link between geometry and algebra, allowing us to calculate distances in terms of coordinates. It also inspired trigonometry.

What did it lead to?

Surveying, navigation, and more recently special and general relativity – the best current theories of space, time, and gravity.